Transient Cornering

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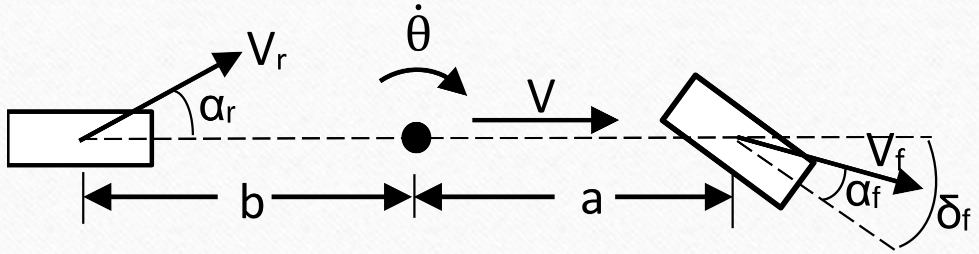
Outline

- Motivation
- Tire Model
- Entire Vehicle Model
- Equations
- Results and Analysis
- Conclusion

Motivation

- Transient is always the case in real world.
- Steady state cornering



- θ: yaw angle
- ϕ: roll angle
- δ_f: steering angle
- V: velocity
- V_f: velocity of the front
- V_r: velocity of the rear
- r: yaw velocity
- α_f: front slip angle
- α_r: rear slip angle
- $C_{\alpha f}$: front cornering stiffness
- $C_{\alpha r}$: rear cornering stiffness
- J_z: moment of inertia about z-axis
- M: roll moment
- a: distance from CG to the front wheel
- b: distance from CG to the rear wheel
- h_r: roll center height
- h_u: unsprung mass height
- F_{zi}: inside tire normal force
- F_{zo}: outside tire normal force
- F_{yi}: inside tire lateral force
- F_{yo}: outside tire lateral force
- F_{rc}: roll center force

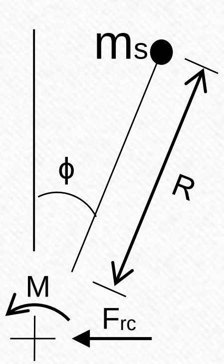
$$C_{\alpha r}\alpha_r + C_{\alpha f}\alpha_f\cos(\delta_f) = M\ddot{y} \quad (1)$$

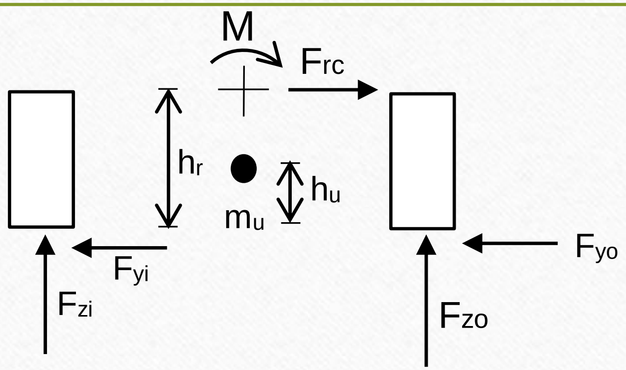
$$bC_{\alpha r}\alpha_r - aC_{\alpha f}\alpha_f\cos(\delta_f) = Jz\ddot{\theta} \quad (2)$$

$$\alpha_r = \tan^{-1}\left(\frac{b\ddot{\theta}}{V}\right) \quad (3)$$

$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{a\ddot{\theta}}{V}\right) \quad (4)$$

$$Jz = 0.9M\left(\frac{a+b}{2}\right)^2 \quad (5)$$





$$F_{rc} = M_s\ddot{y} \quad (6)$$

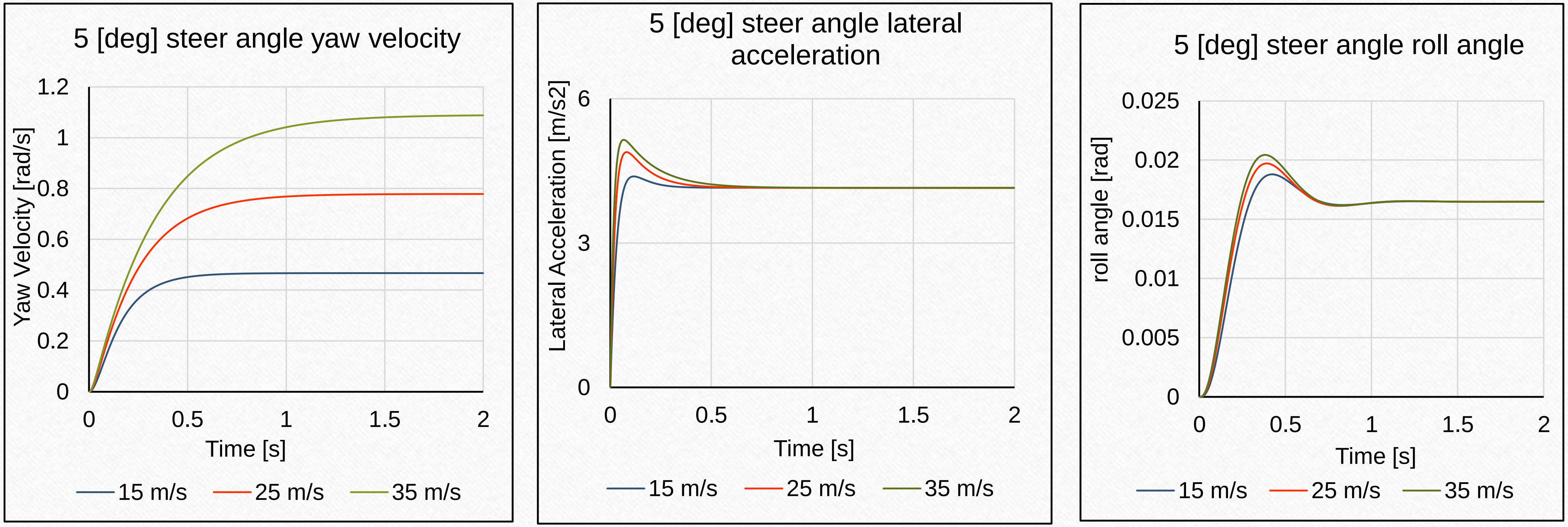
$$M - F_{rc}R\cos\phi - M_s g R\sin\phi + c\dot{\phi} = J_s\ddot{\phi} \quad (7)$$

$$M = k_\phi\phi \quad (8)$$

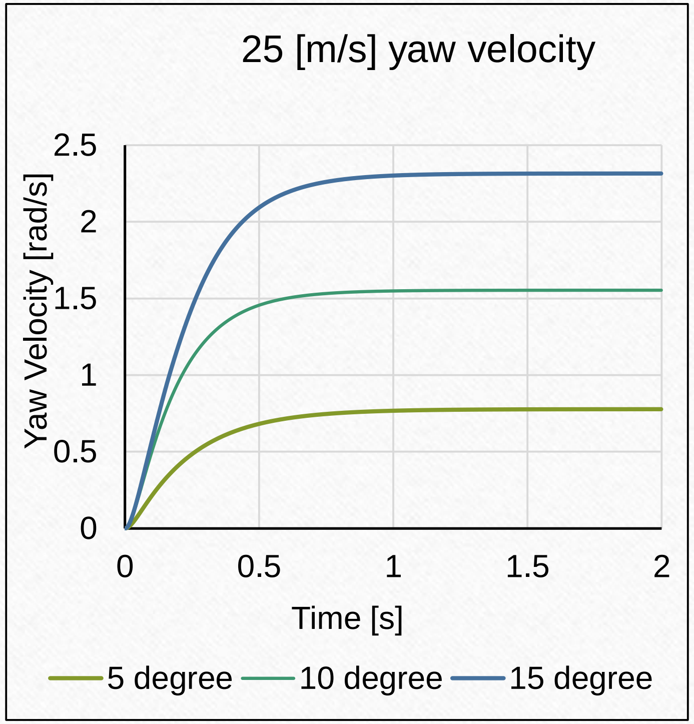
Assuming no roll of unsprung mass:

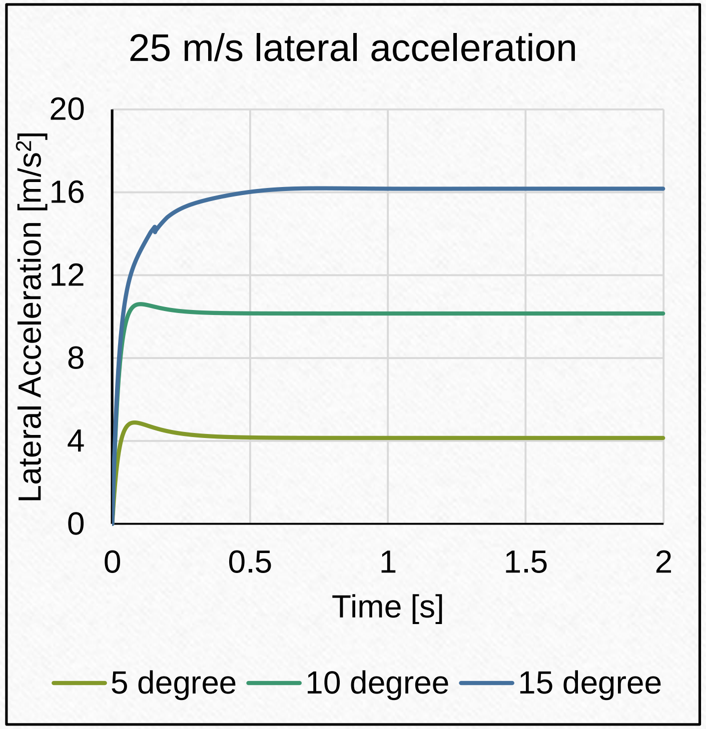
$$(F_{zo} - F_{zi})\frac{t}{2} - h_c(F_{yi} + F_{yo}) - (h_r - h_u)F_{rc} - M - c\dot{\phi} = 0 \quad (9)$$

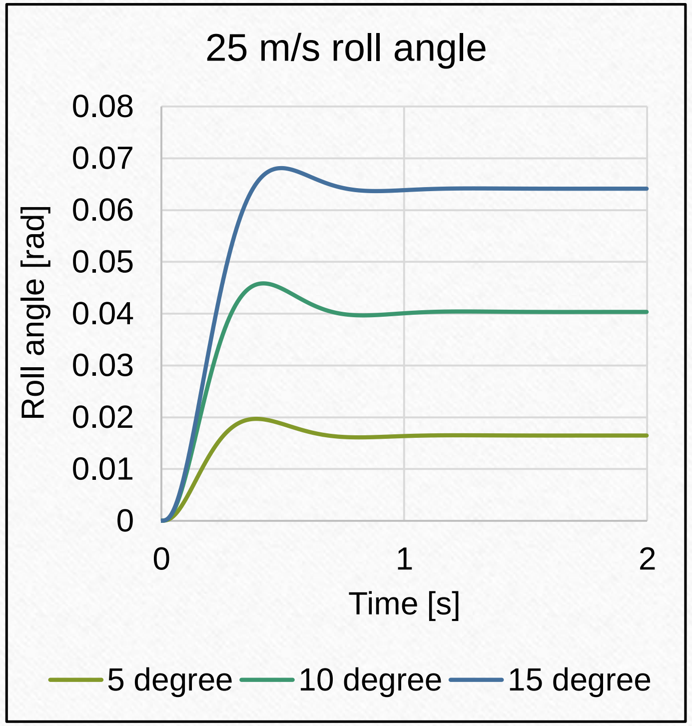
Same Steer Angle Different Speed



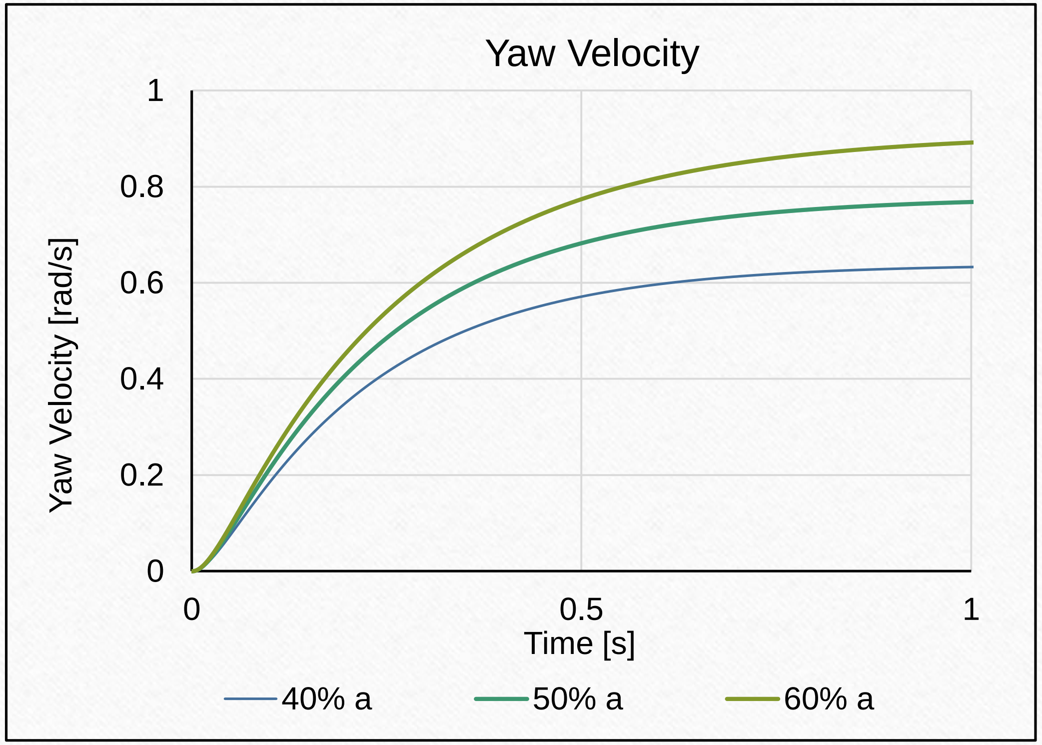
Same Speed Different Steer Angle

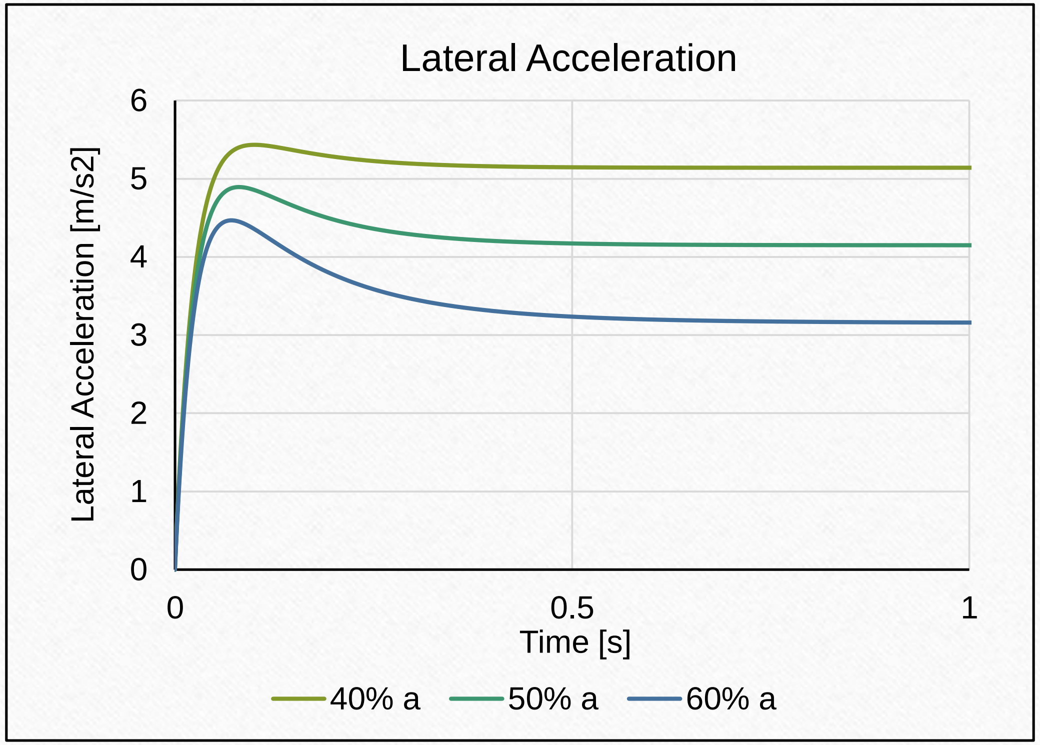




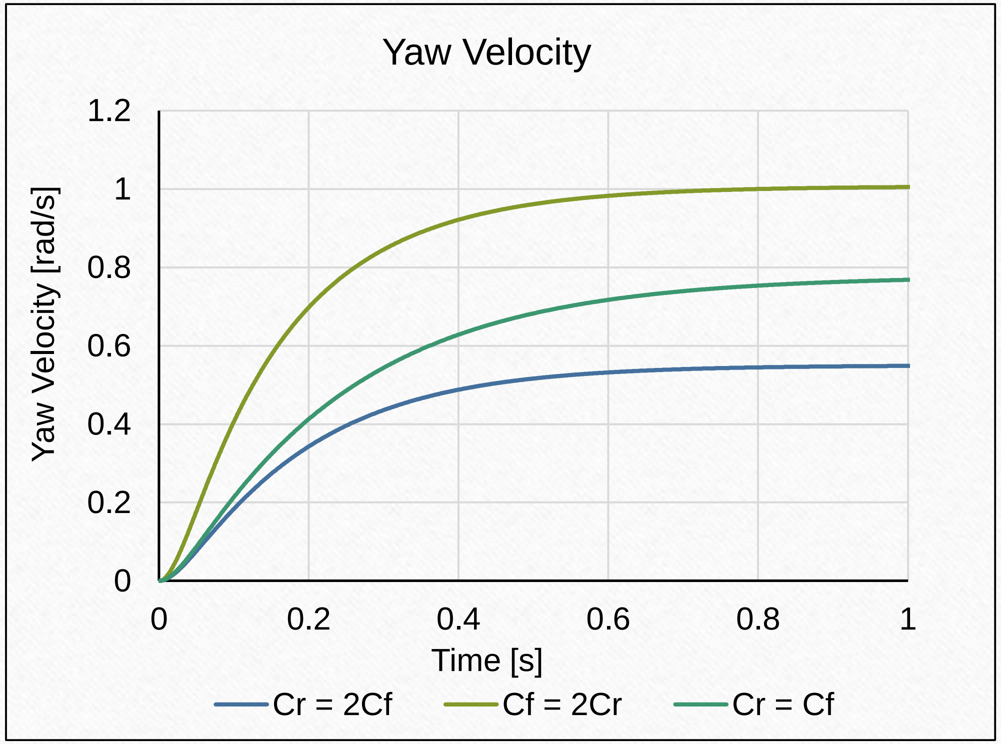


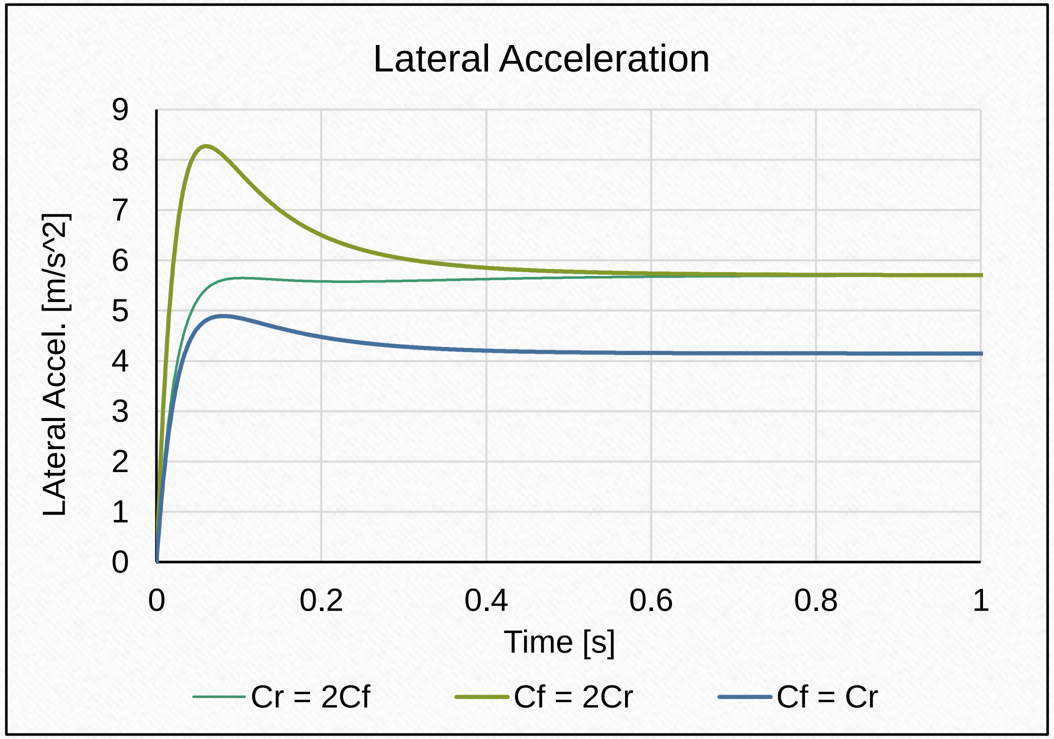
Different Weight Distributions (a)



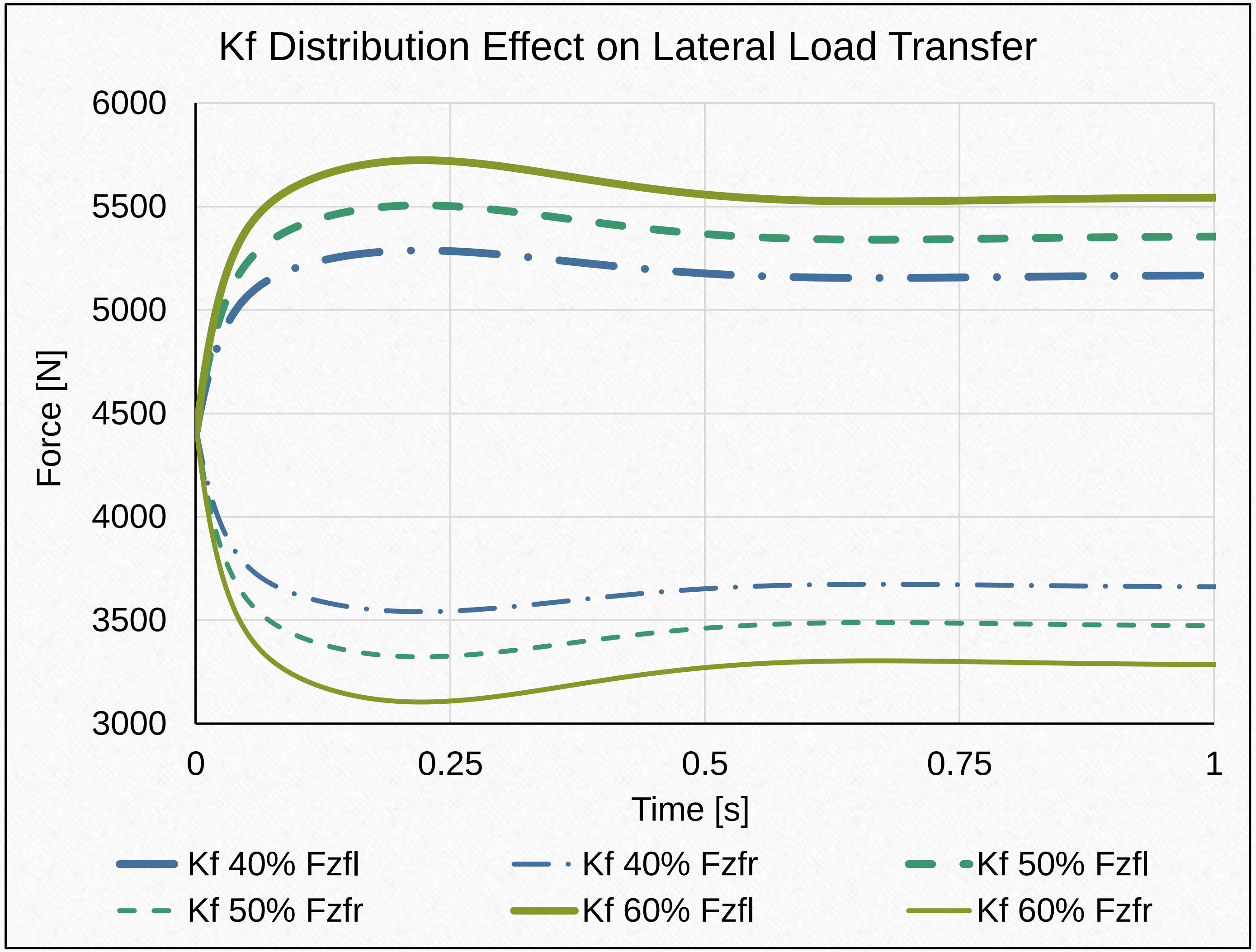


Different Cornering Stiffness

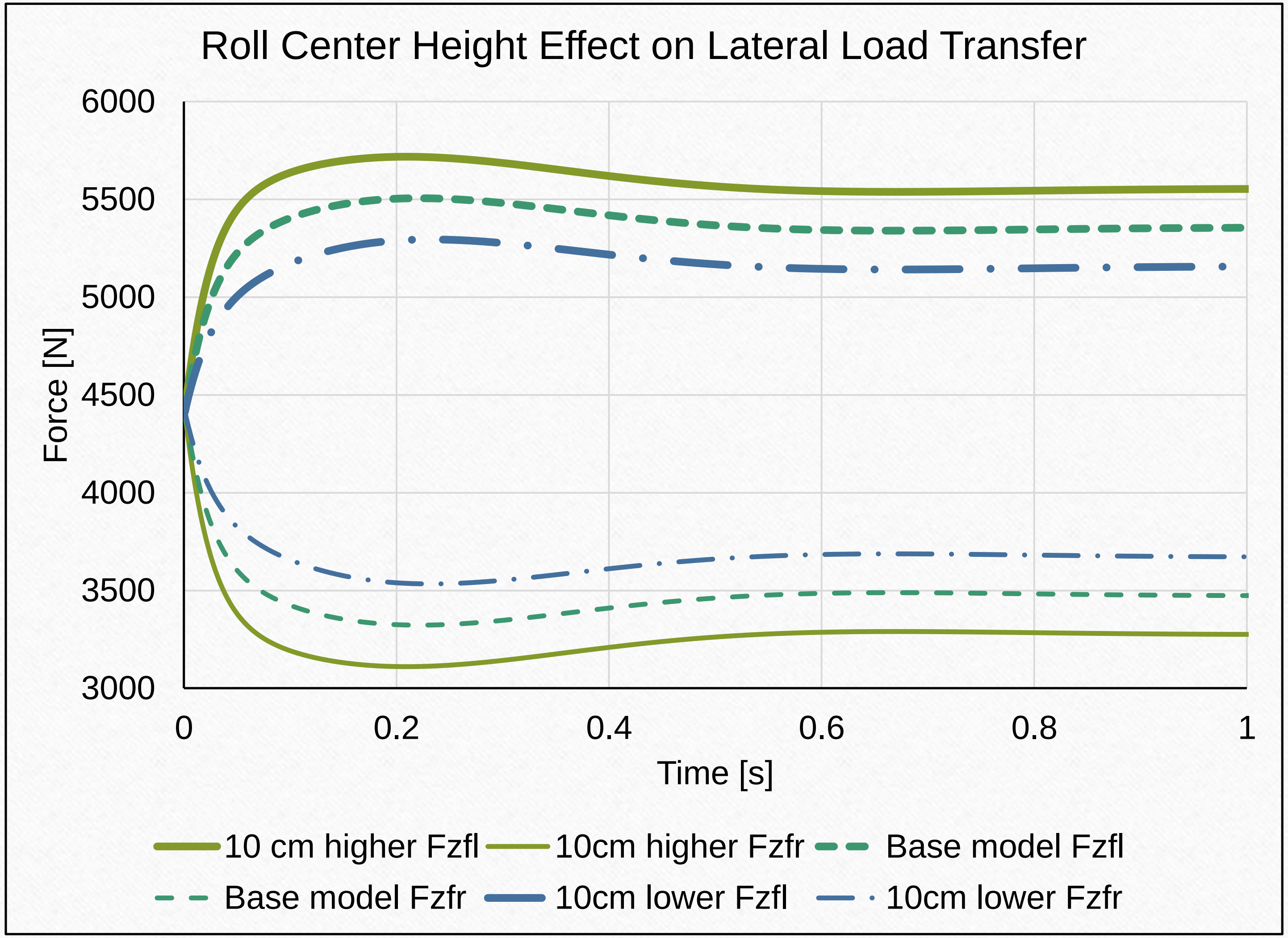




Different roll stiffness distribution



Different Roll Center Heights



Tire Analysis

- Relaxation Length
- Single Point Contact Model
- Derivation
- Results
  - Sideslip Angle
  - Camber Angle

Effective Relaxation Length

- The distance the tire has to travel to yield 63.2% of the steady-state force after a step input.

$$F_y = C_\alpha \cdot \alpha \cdot (1 - e^{-t/\tau})$$

$$(1 - e^{-t/\tau}) = 1 - e^{-1}$$

Procedure for analytical solution of transient slip quantities

Slip Velocities

Longitudinal Velocity of Wheel Center

Initial values for deflections

Side slip angle (α)

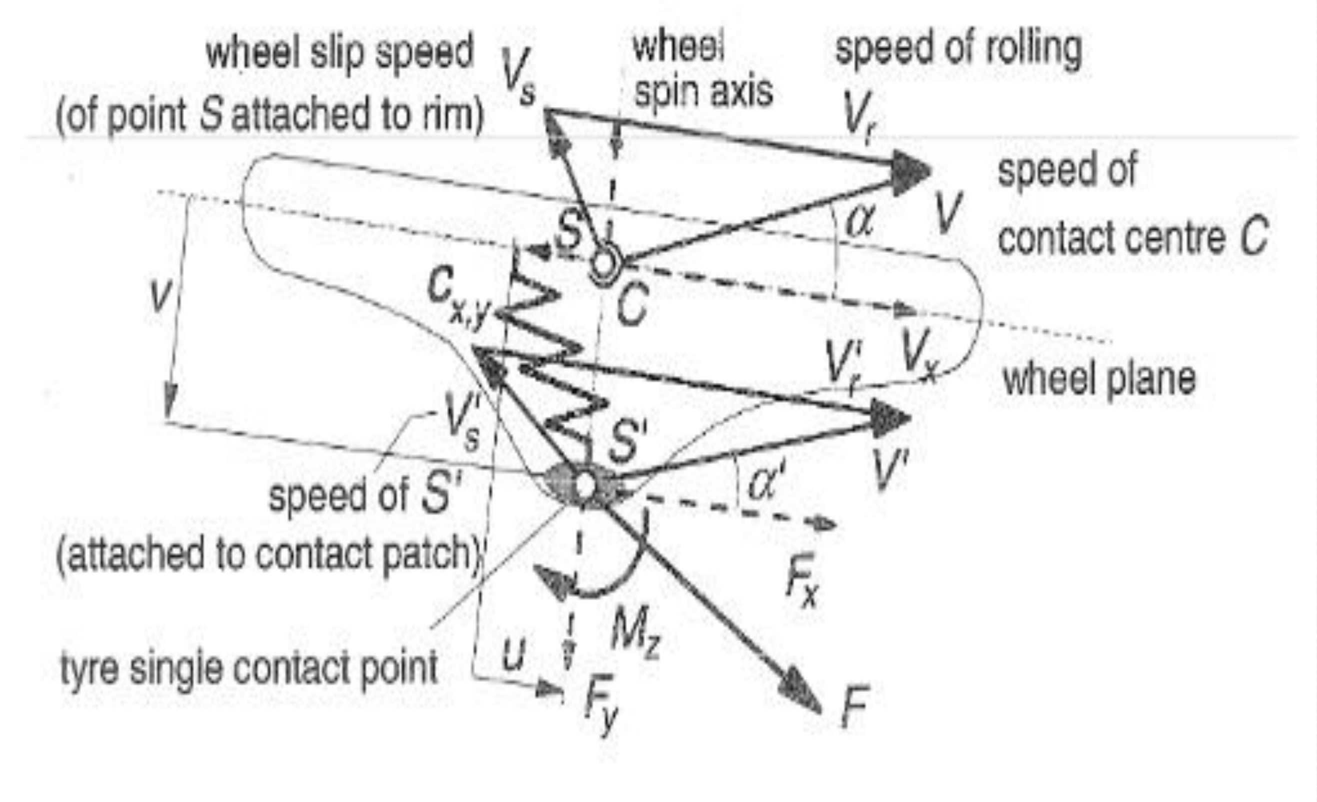
Camber angle (γ)

Deferential Equation

Contact point deflections
u, v

Transient slip quantities
α' ,κ',γ'

Single Point Contact Patch Model



Derivation to find v_α and v_γ

1) $V_{sy} = |V\_x|\alpha$

2) $V_{sx} = (V_x - \omega r_e)/(V_x)$

$$\frac{du}{dt} = -(V_{sx} - V'_{sx})$$

$$\frac{dv}{dt} = -(V_{sy} - V'_{sy})$$

$$\alpha' \approx \frac{v_\alpha}{\sigma_\alpha}$$

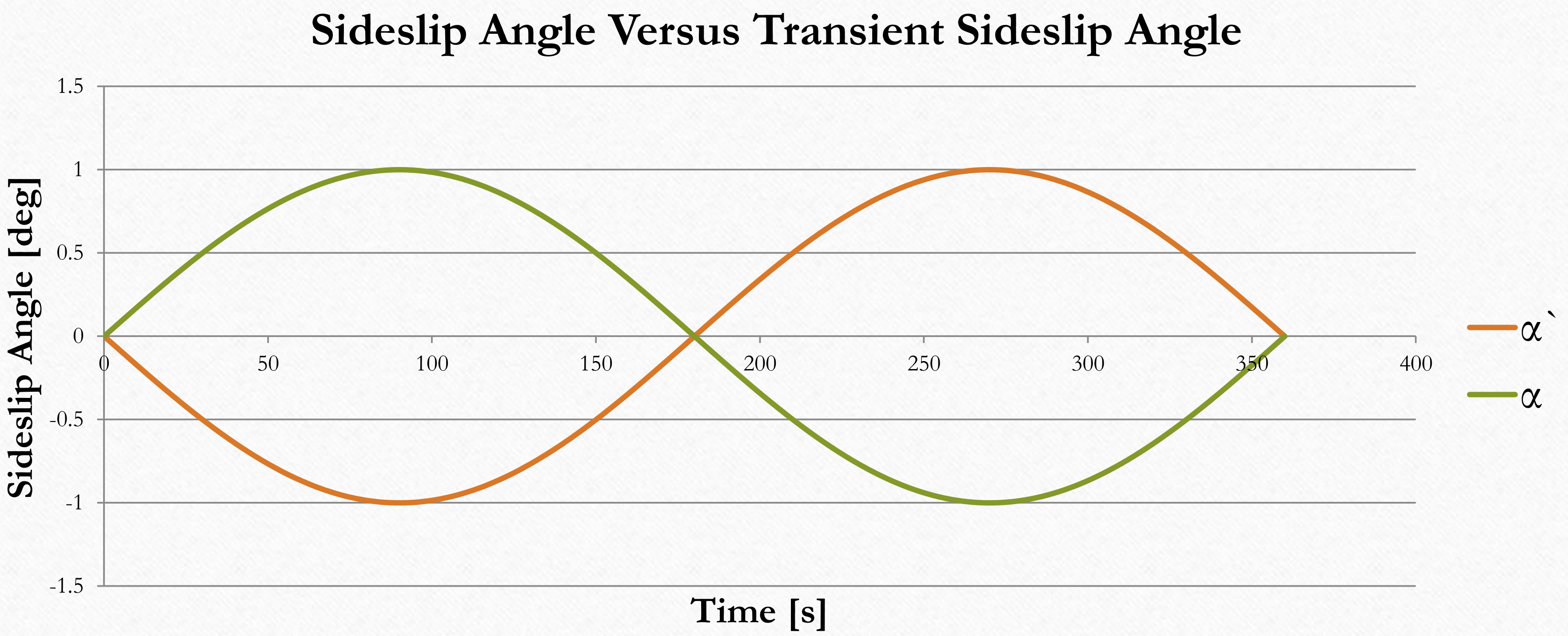
$$\gamma' = \frac{C_{F\alpha} v_\gamma}{C_{F\gamma} \sigma_\alpha}$$

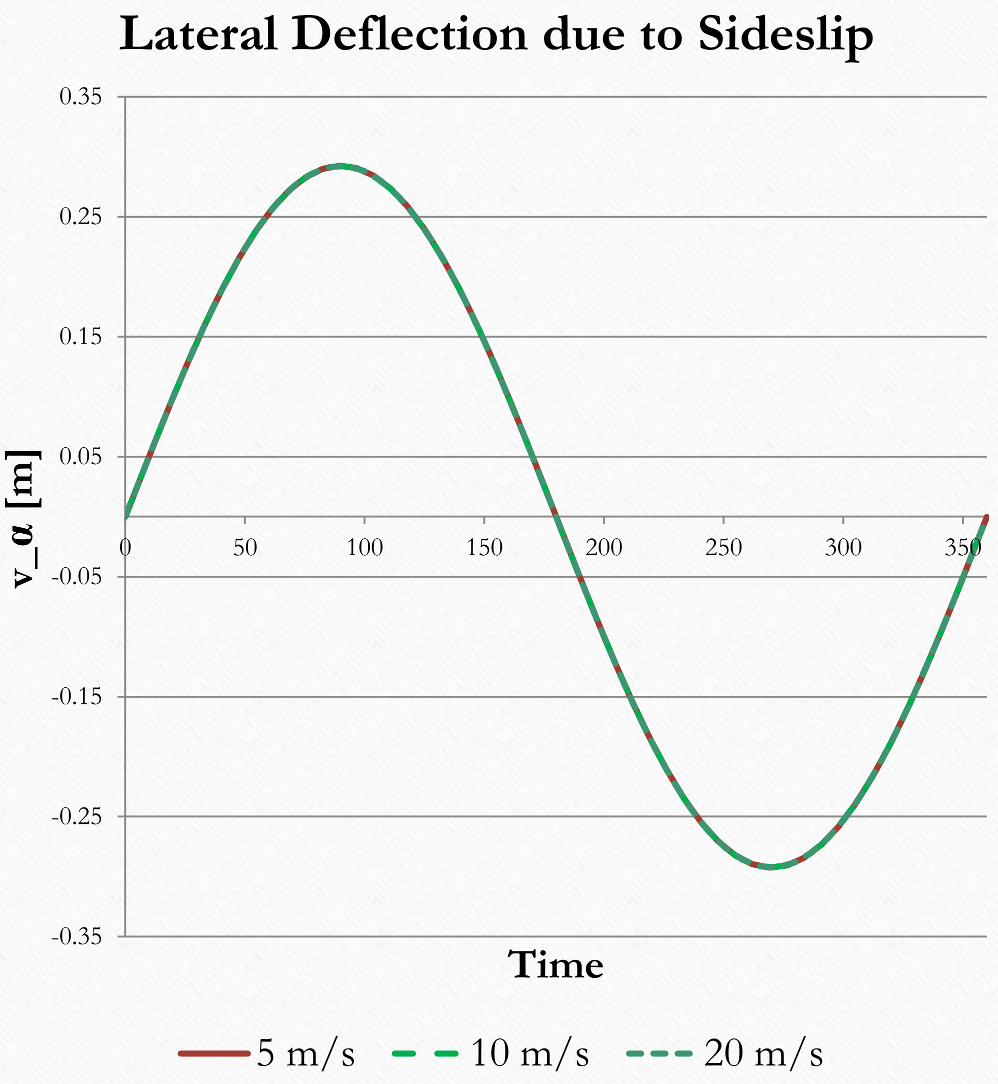
$$\frac{dv_\alpha}{dt} + \frac{1}{\sigma_\alpha}|V_x| v_\alpha = |V_x|\alpha = -V_{sy}$$

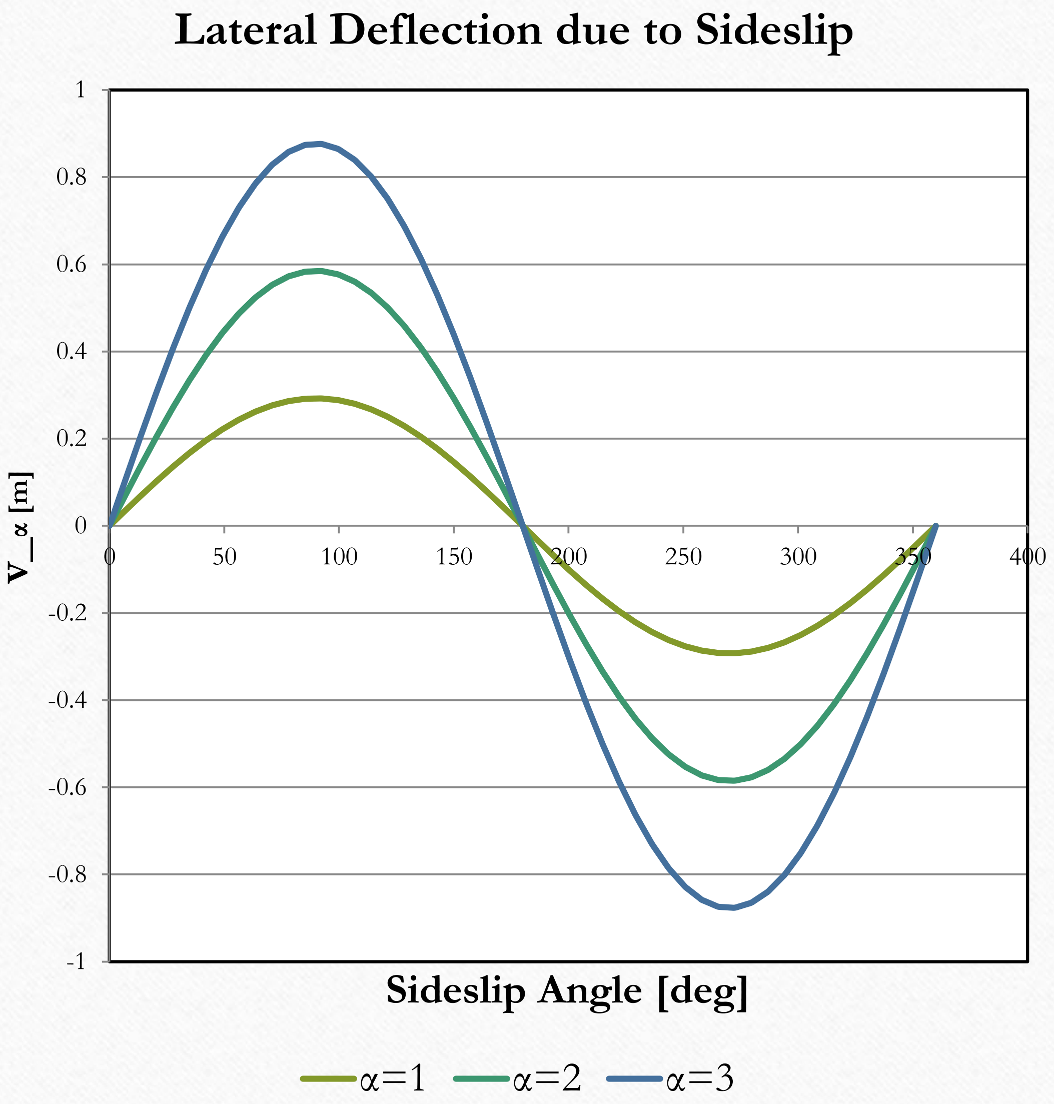
$$\frac{dv_\gamma}{dt} + \frac{1}{\sigma_\alpha}|V_x| v_\gamma = \frac{C_{F\gamma}}{C_{F\alpha}} |V_x| \gamma$$

```
t=t_1
alpha=h*sin(t_1)
Vel_x=20 [m/s]                                          "velocity of vehicle"

{gamma=h*sin(t_1)}

{"alpha=1 [deg]"                                        "lateral slip angle"
kappa= .05 [-]                                          "longitudinal slip ratio; 1.05"}
gamma=h*sin(t_1)                                        "camber angle"

{V_wheel=Vel_x*(1+kappa)}
{alpha_rad=alpha*convert(deg,rad)}

"stiffnesses"
C_F_alpha=850*convert(N/deg,N/rad)                      "sideslip cornering stiffness"
C_F_kappa=1500000*convert(N/deg,N/rad)                  "longitudinal slip stiffness"
C_F_gamma=36*convert(N/deg,N/rad)                       "camber stiffness"
C_F_x=435000 [N/m]                                      "longitudinal tire stiffness"
C_F_y=166500 [N/m]                                      "lateral tire stiffness"

"slip velocities"
{V_sx=Vel_x-V_wheel}                                    "longitudinal velocity of contact patch; ACCELERATION; assume wheel velocity is 5% greater than actual velocity"
{V_sy=Vel_x*alpha}                                      "sideslip velocity of contact patch"

"relaxation lengths"
sigma_alpha=C_F_alpha/C_F_y                             "relaxation length for longitudinal slip"
"sigma_kappa=C_F_kappa/C_F_x"                           "relaxation length for sideslip"

"outputs"

{"diff. eqn for finding lateral deflection due to sideslip"
v`_alpha+(1/sigma_alpha)*Vel_x*v_alpha=Vel_x*alpha
v_alpha=integral(v`_alpha, t,0,360)
alpha`=v_alpha/sigma_alpha                              "transient lateral slip angle"}

"diff. eqn for finding lateral deflection due to camber angle"
v`_gamma+(1/sigma_alpha)*Vel_x*v_gamma=(C_F_gamma/C_F_alpha)*Vel_x*gamma
v_gamma=integral(v`_gamma, t,0,360)
gamma`=(C_F_alpha*v_gamma)/(C_F_gamma*sigma_alpha)"transient camber angle"
```

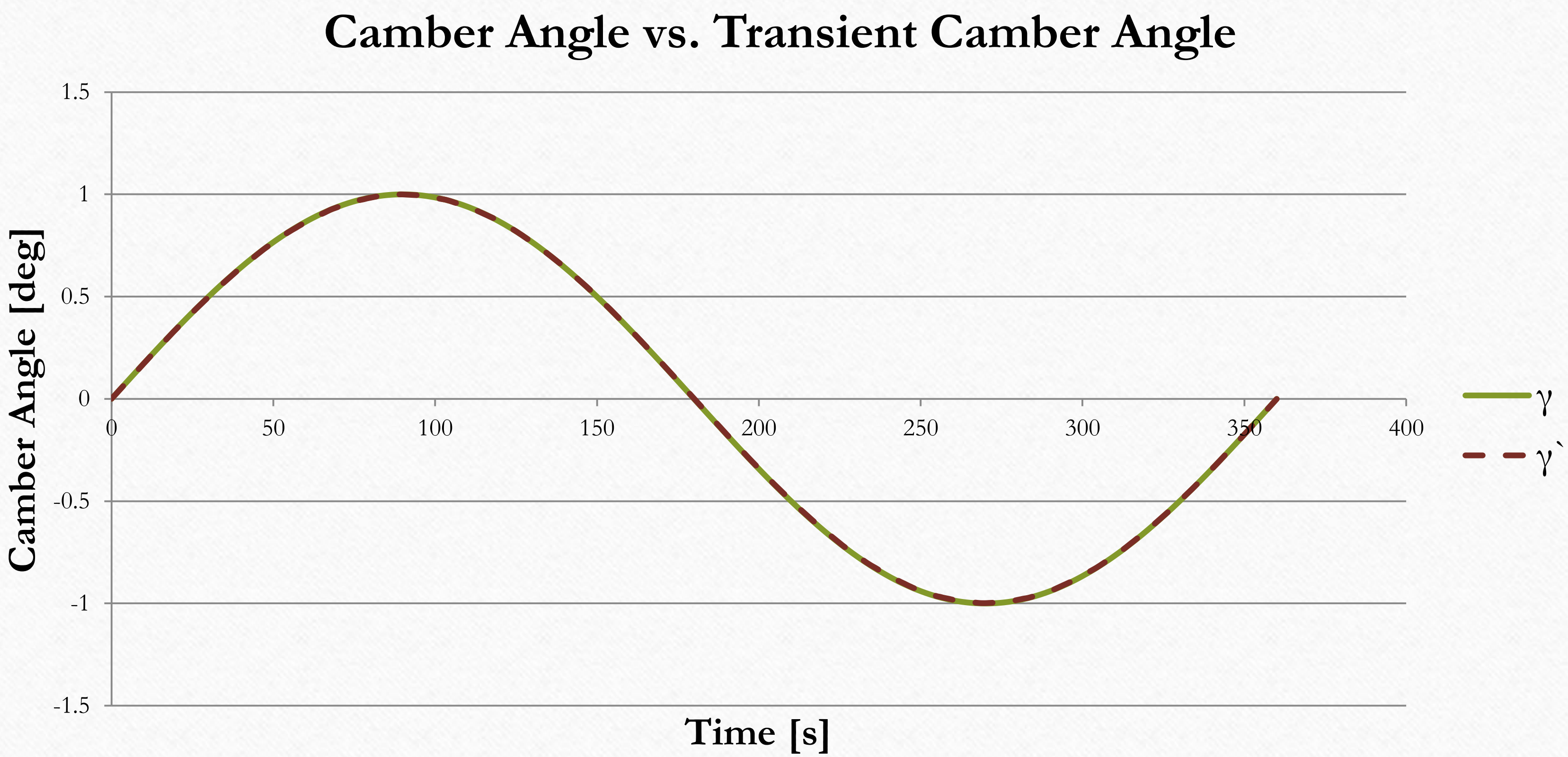
Results- Sideslip Angle

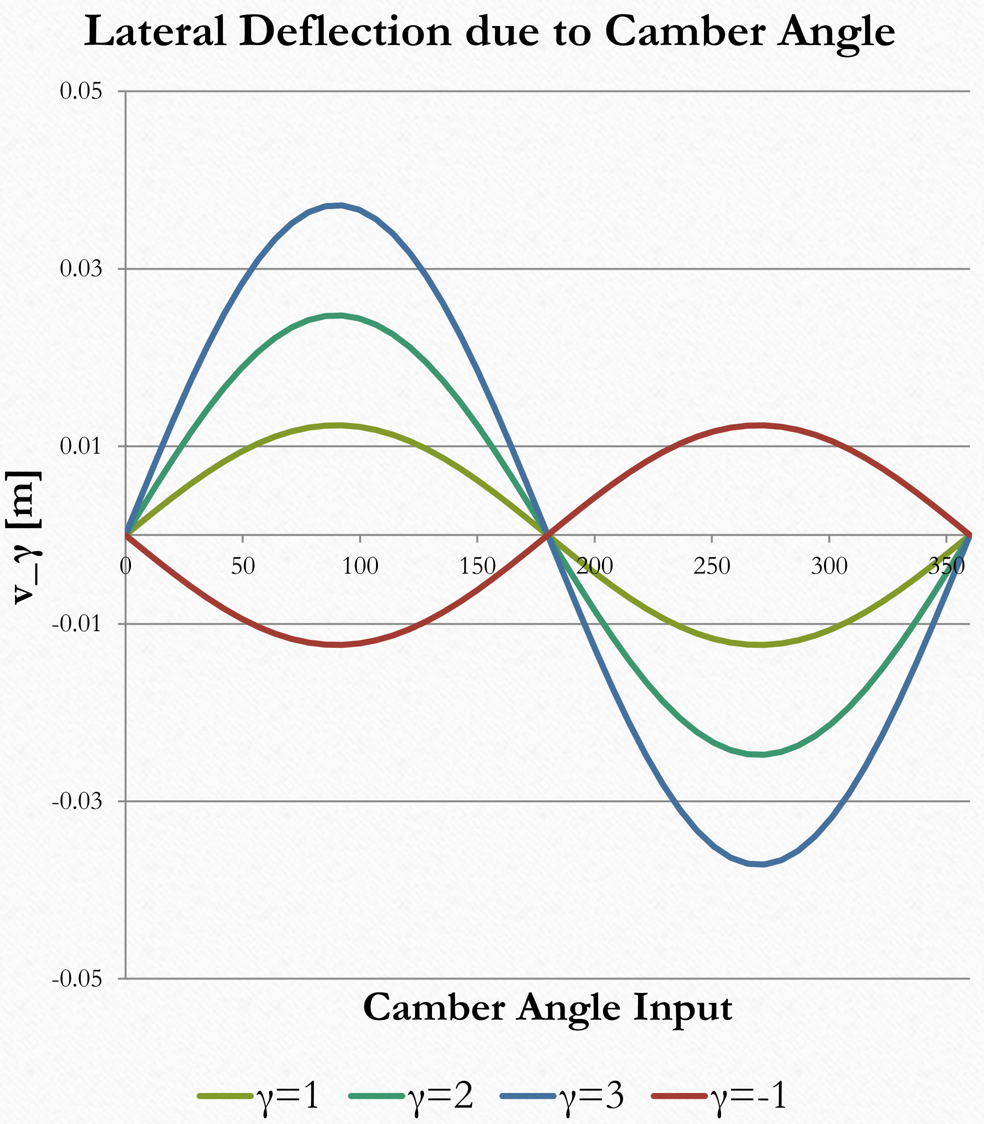


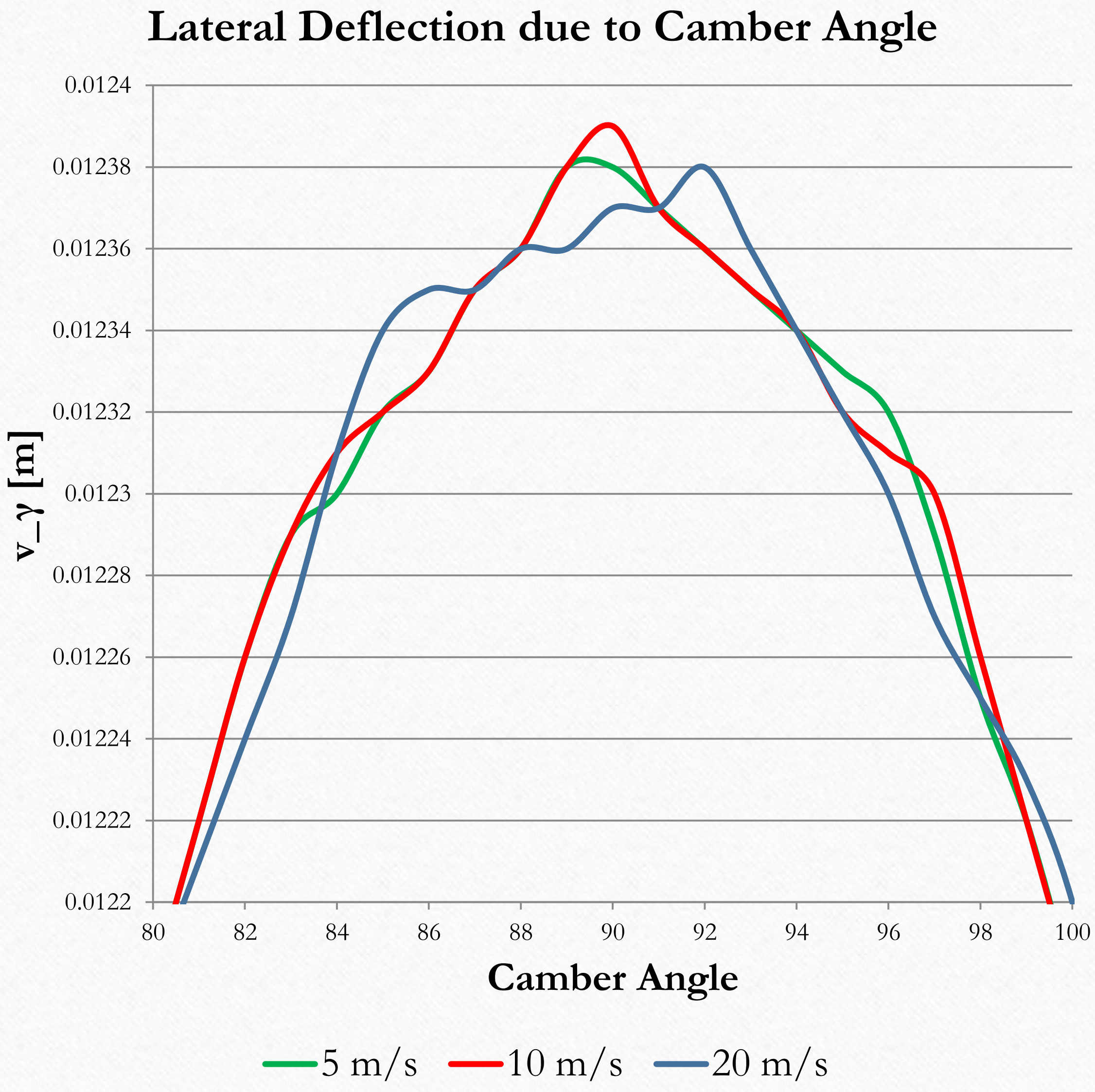

Lateral Deflection due to Sideslip
0.35
0.25
0.15
0.05
-0.05
-0.15
-0.25
-0.35
0
50
100
150
200
250
300
350
v_α [m]
Time
5 m/s
10 m/s
20 m/s


Lateral Deflection due to Sideslip
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
50
100
150
200
250
300
350
400
V_α [m]
Sideslip Angle [deg]
α=1
α=2
α=3

Results- Camber Angle




Lateral Deflection due to Camber Angle
0.05
0.03
0.01
-0.01
-0.03
-0.05
0
50
100
150
200
250
300
350
v_γ [m]
Camber Angle Input
γ=1
γ=2
γ=3
γ=-1


Lateral Deflection due to Camber Angle
0.0124
0.01238
0.01236
0.01234
0.01232
0.0123
0.01228
0.01226
0.01224
0.01222
0.0122
80
82
84
86
88
90
92
94
96
98
100
v_γ [m]
Camber Angle
5 m/s
10 m/s
20 m/s

Conclusion and Questions